

LAGRANGIAN MODELS OF PARTICLES WITH SPIN: THE FIRST SEVENTY YEARS ¹

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Abstracts: We briefly review models of relativistic particles with spin. Departing from the oldest attempts to describe the spin within the lagrangian framework we pass through various non supersymmetric models. Then the component and superfield formulations of the spinning particle and superparticle models are reviewed. Our focus is mainly on the classical side of the problem, but some quantization questions are mentioned as well.

1 Introduction

The aim of the present brief review is to indicate some essential aspects of the theory of relativistic point particle with spin. The selected models are presented mostly historically as they were appearing, to show the development of ideas in the period of 70 years - starting from the very beginning.

The first published work concerning the lagrangian description of the relativistic particle with spin was the paper by Frenkel which appeared in 1926. In that time main considerations go towards the derivation of the equations of spin precession in the external electromagnetic field. Then to its relativistic generalization. The last one was achieved by Bargmann, Michael and Telegdi [3] 33 years after the Frenkel model was constructed. However only the work of Frenkel contains the Lagrangian defining the model of a particle, not only a considerations on the equations of motion level. Then there is a forty years long gap in the activity in constructing such a models. However, in the mean time - in the fifties, the idea of anticommuting coordinates

¹Dedicated to Jurek Lukierski on the occasion of his 60th anniversary

emerges in works of Martin [19], Matthews and Salam [20], and Tobočan [21]. Later it strongly influenced particle models [44, 29].

The silence was broken with the work of Barut [4]. Then in early seventies wider interest in the subject begins with the works of Hanson, Regge [5]; Grassberger [7]; Casalbuoni [13]; Berezin, Marinov [17]. Some of these models involve anticommuting coordinates. Further growth of the interest was stimulated by dynamically developing research in the supersymmetry and supergravity and then superstrings theory with the wide use of \mathbf{Z}_2 -graded structures. This period lasts from the eighties to the present decade with such a new models: Brink-Schwarz [27], Brink-diVecchia-Howe [25], de Azcárraga-Lukierski [23], Siegel [31, 32], Volkov-Soroka-Tkach [67]. Above mentioned models fall in principle into different categories. The classification can be made due to the such attributes as mass, algebraical (conventional or anti-commuting) and geometrical character of the internal degrees of freedom (vectorial, spinorial, twistorial).

In the sequel we shall adopt the following naming conventions. Models involving only conventional coordinates will be called the classical models. They in principle are of vectorial or tensorial type. The models involving anti-commuting coordinates are generally called here pseudoclassical. These with the anti-commuting vectorial degrees of freedom are called the spinning particles and these with the spinorial anti-commuting degrees of freedom are called superparticles.

The type of extension of the configuration space of the relativistic particle by the commuting or anti-commuting coordinates determines the symmetry and the behaviour of the model in the external field and upon quantization. Starting demand is to have object which is at least Poincaré invariant. Classical vectorial particles and spinning particles couple properly to the external fields, however only the latter ones can be correctly quantized and do not give undesirable classical selfacceleration and effect of Zitterbewegung type. Spinning particles are in some sense the classical limit of the Dirac particle. After the first quantization these new anti-commuting variables are mapped into the Dirac matrices and they disappear from the theory. This is a general feature of the spinning particle models.

On the other hand the extension of the configuration space by the anti-commuting spinorial variables yields the models which are super-Poincaré invariant. In contrast to the spinning particle models their first quantization gives theory which still involves the anti-commuting variables. As a result of

quantization we get rather not a single quantum particle with spin $\frac{1}{2}$ but a minimal supermultiplet.

The organization of the review follows the models classification sketched above. We begin with the two principal categories of the classical models and the so called arbitrary spin particles. Then the pseudoclassical group of models is presented including spinning particles, super-particles, twistorial and harmonic particles, arbitrary superspin models. Next we comment the double supersymmetric models with the spinning superparticle in the component and superfield form. We conclude this brief review recapitulating some new developments including first attempts of q-deformation the relativistic model of the spinning particle and the κ -relativistic model of a particle.

The literature on the particle with spin is vast. We include here, only the very selective list of references.

2 Classical models

In this section I shall briefly present the classical models of the relativistic particle with spin. The adjective classical means here not only that a model is not quantum but also that it is described by means of the commuting variables only.

2.1 Vectorial models

Historically the first model has been introduced by Frenkel. The spin of the particle in his model is described directly by a tensor of spin $S_{\mu\nu}$, which is assumed to be proportional to the tensor of internal magnetic moments $M_{\mu\nu}$. It enters the lagrangian via the "transversality condition"

$$S_{\mu\nu}\dot{x}^\nu = 0. \quad (1)$$

to reduce the number of independent degrees of freedom. We shall call it the Frenkel condition. Explicit form of the action is as follows

$$S = \int d\tau (\lambda \dot{x}^2 + a^\mu S_{\mu\nu} \dot{x}^\nu + S_{\mu\nu} \omega_{\mu\nu}), \quad \omega_{\mu\nu} = -\omega_{\nu\mu} \quad (2)$$

It yields the equations of the motion of the form

$$\dot{S}_{\mu\nu} - (\dot{x}_\mu S_{\nu\rho} - \dot{x}_\nu S_{\mu\rho}) a^\rho = 0 \quad (3)$$

$$(\lambda x_\mu + S_{\mu\rho} a^\rho)^\cdot = 0 \quad (4)$$

Some developments of this model were done 33 years later by Barut. To describe internal degrees of freedom he introduces the frame of four fourvectors $q_{(i)}^\mu$ $i = 0, 1, 2, 3$; such that $q_{(0)}^\mu$ is proportional to (\dot{x}^μ) and the rest is orthogonal to (\dot{x}^μ) . Using implicit form of the action

$$S = \int d\tau L(\dot{x}^\mu, q_{(i)}^\mu, \dot{q}_{(i)}^\mu) \quad (5)$$

and the following definition of the tensor of spin

$$S_{\mu\nu} = \frac{\partial L}{\partial \dot{q}_{(i)}^\mu} q_{(i)}^\nu - \frac{\partial L}{\partial \dot{q}_{(i)}^\nu} q_{(i)}^\mu \quad (6)$$

he gets the following form of the equations of motion

$$\dot{p}^\mu = 0 \quad (7)$$

$$\dot{S}_{\mu\nu} + p_{[\mu} x_{\nu]} = 0 \quad (8)$$

Moreover the Frenkel condition is valid.

Historically the next classical model was of different kind. It was an exemplification of the idea that particle is an irreducible representation of the Poincaré group. Namely in the model of Hanson and Regge the configuration space has coordinates $(x_\mu, \Lambda_{\mu\nu})$, where $\Lambda \in L^\uparrow$ (L^\uparrow - orthochronous Lorentz group). It turns out that dependence of the lagrangian function on the \dot{x} , Λ , $\dot{\Lambda}$ should be restricted to $L(\dot{x}^\mu, \sigma^{\mu\nu})$, where $\sigma^{\mu\nu} = \Lambda^{\lambda\mu} \dot{\Lambda}_\lambda^\nu$. Now the tensor of spin is given by the formula

$$S^{\mu\nu} = \frac{\partial L}{\partial \sigma^{\mu\nu}} \quad (9)$$

and the equations of motion take the same form as in the Barut model. The demand, that in the non-relativistic limit the particle has only three spin degrees of freedom is realized by the condition

$$S_{\mu\nu} p^\nu = 0 \quad (10)$$

It was originally introduced by Dixon [24]. In this model it should be included into the action. Let us note that the Frenkel condition and Dixon condition yield the essential differences in possible motions of particles, even in the free case. Explicit realization of such an action takes the form

$$L(\dot{x}^\mu, \sigma^{\mu\nu}) = -\frac{1}{\sqrt{2}} \{ A\dot{x}^2 - B\sigma^2 + [(A\dot{x}^2 - B\sigma^2)^2 \quad (11)$$

$$- 8B(A\dot{x}\sigma^2\dot{x} - 2B\det\sigma)]^{\frac{1}{2}} \}^{\frac{1}{2}}, \quad (12)$$

where $\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu}$ and $\dot{x}\sigma^2\dot{x} = \dot{x}_\mu\sigma^{\mu\nu}\sigma_{\nu\lambda}\dot{x}^\lambda$. The constraints are of the form

$$S_{\mu\nu}p^\nu = 0 \quad (13)$$

$$p^2 - \frac{A}{4}S_{\mu\nu}S^{\mu\nu} = 0 \quad (14)$$

The last constraint follows from the reparametrization invariance of the action. The mass of the particle is renormalized here by the square of the $S^{\mu\nu}$. This model does not give after the first quantization the Dirac particle. Along similar lines is constructed the BMSS model proposed in Ref. [9]. Here again the particle with spin is directly tied up to the irreducible representations of the Poincaré group P_+^\uparrow . To this end, as a configuration space one takes $(z^\mu, \Lambda) \in P_+^\uparrow$. The matrix is decomposed into the momentum and spin tensor, where

$$p_\mu = m\Lambda_{\mu 0}, \quad m0 \quad (15)$$

$$S = i\lambda\Lambda\sigma_{12}\Lambda^{-1}, \lambda \in \Re \quad (16)$$

$$(\sigma^{\mu\nu})_{\rho\lambda} = -i\delta_{\rho\lambda}^{\mu\nu} \quad (17)$$

This means that

$$S_{\mu\nu} = \lambda(\Lambda_{\mu 1}\Lambda_{\nu 2} - \Lambda_{\mu 2}\Lambda_{\nu 1}) \quad (18)$$

and by the construction

$$S_{\mu\nu}S^{\mu\nu} = 2\lambda^2 \quad (19)$$

$$p^\mu S^{\mu\nu} = 0 \quad (20)$$

Therefore the model written on such a space has to be of Dixon category. The lagrangian finally defining the model is taken in the following form

$$L = p_\mu \dot{z}^\mu + \frac{i\lambda}{2} \text{Tr}(\sigma_{12}\Lambda^{-1}\dot{\Lambda}) \quad (21)$$

The resulting equations of motion are the same as in the Hanson-Regge model. The four-momenta and four-velocities are related in the standard way. This type of model has been recently reformulated and a correspondence to the pseudoclassical model was proposed [10].

Now let us come back to the vectorial models. In 1978 Grassberger proposed the description [7] in which the Minkowski space is extended by the two

four-vectorial internal degrees of freedom $(x_\mu) \mapsto (x_\mu, a_\mu, b_\mu)$. The Poincaré invariant action is defined by means of the lagrangian

$$L = \frac{1}{2}m(1 - \dot{x}^2) + \dot{x}_\mu(\beta b^\mu - \alpha a^\mu) + \frac{1}{2}(\dot{b}_\mu a^\mu - \dot{a}_\mu b^\mu) \quad (22)$$

The Lagrange multipliers m , α , β are introduced to provide the necessary constraints

$$\dot{x}^2 = 1 \quad (23)$$

$$a_\mu \dot{x}^\mu = 0 \quad (24)$$

$$b_\mu \dot{x}^\mu = 0 \quad (25)$$

The tensor of spin obtained from the above lagrangian is composed of the new vectorial internal co-ordinates i.e.

$$S_{\mu\nu} = a_\mu b_\nu - a_\nu b_\mu \quad (26)$$

and it obeys the Frenkel condition with $S_{\mu\nu}S^{\mu\nu} = \text{const.}$ Now there are some new features in the equations of motion of this model, namely

$$\frac{d}{d\tau}(m\dot{x}^\mu + \eta^\mu) = 0 \quad (27)$$

$$\dot{S}_{\mu\nu} + p_{[\nu}\dot{x}_{\mu]} = 0, \quad \eta_\nu = \alpha a_\nu + \beta b_\nu \quad (28)$$

This means that if such a particle is coupled to the external electromagnetic field, or has only passed through the bounded area with non-vanishing field, due to the presence of η_ν term the center of mass and the center of the charge need not coincide.

Five years later Cognola, Soldati, Vanzo and Zerbini [8] proposed another vectorial model. Its configuration space is the same as for the Grassberger model, but the new lagrangian takes the form

$$L = -m^2(P_{\mu\nu}\dot{x}^\mu\dot{x}^\nu)^{\frac{1}{2}} - \dot{a}_\nu b^\nu \quad (29)$$

with $S_{\mu\nu}$ given by eq.(26) and

$$P_{\mu\nu} = g_{\mu\nu} - \frac{S_{\mu\rho}S_\nu^\rho}{S^2} = g_{\mu\nu} - Q_{\mu\nu} \quad (30)$$

The constraints are now of the form

$$p^2 = m^2, \quad p_\mu = \frac{m}{\sqrt{P_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}}(\dot{x}_\mu - Q_{\mu\nu}\dot{x}^\nu) \quad (31)$$

$$p_\mu a^\mu = 0 \quad (32)$$

$$p_\mu b^\mu = 0 \quad (33)$$

what obviously means that the Dixon condition is fulfilled. The internal degrees of freedom are here orthogonal to the momenta, moreover the direction of fourvelocity and fourmomenta can be different. Equations of motion take the standard form.

Another branch of the models of the classical spinning particle is connected with the Souriau's notion of the space of motions [52] and the coadjoint orbit method. As a sample we indicate here the Refs. [52, 54, 55] and recently [53].

The classical models sketched in this section have one property in common, they do not give after the first quantization the accepted quantum relativistic Dirac particle. On the other side, coupled to the external electromagnetic field, in the limit of the weak homogenous field, they yield the Bargmann-Michel-Telegdi equations. The models fulfilling the Frenkel condition have helisoidal curves as the solutions of the equations of motion. This can be interpreted as a counterpart of the Zitterbewegung solution for the Dirac particle, however from the classical point of view such a trajectory for a free particle can be hardly accepted.

Despite the technical subtleties of different models of this kind the behaviour of the particular type of the particle with spin depends mainly on the type of the "orthogonality condition" for the internal degrees of freedom i.e. the Frenkel or the Dixon condition. The latter one seems to be more natural.

2.2 Spinorial models (arbitrary spin particles)

Finally let us comment classical models of the particle with spin described by the spinorial coordinates. The presence of the commuting spinor not necessarily means that model describes the particle with spin [58, 57]. In the twistor-like approach the massless point particle for example, has the action of the form

$$S = \int d\tau p_m (\dot{x}^m - \bar{\lambda} \gamma^m \lambda), \quad (34)$$

where p_m is the particle momentum and λ^α is a commuting spinorial variable, needed to ensure the mass shell condition (another interesting spinorial model has been discussed in Refs.[64] and [65] with the action of the form $S = \int d\tau \bar{\lambda} \gamma_m \lambda \dot{x}^m$).

The above action is a good starting point to supersymmetric generalizations. There are models of point particles with spin described by commuting spinors, however in such an approach not only the spin one half appears but also the whole spectrum of spin values. This justifies the name: arbitrary spin particles [61, 62, 63, 59, 60]. As an illustration let us consider two models of the arbitrary spin massive particles. The lagrangian of the first model [59] is closely related to the spinning superparticle model [73], and is given in the form

$$L = \frac{1}{2}(e^{-1}\dot{x}^2 + em_0^2) - h\dot{x} \cdot j + 2\bar{\eta}\dot{\eta}, \quad (35)$$

where $h, m_0 \in \mathfrak{R}_+$ and η is the Majorana spinor. The current $j^a = \bar{\eta}\gamma^a\eta$ has vanishing square. The resulting equation of motion have the form

$$\frac{d}{dt}(e^{-1}\dot{x} - hj) = 0 \quad (36)$$

$$\dot{x}^2 + m_0^2 c^2 = 0 \quad (37)$$

$$h\dot{x}\eta - 2\dot{\eta} = 0 \quad (38)$$

Obviously the conserved angular momentum tensor has a contribution from the "internal" degrees of freedom

$$M_{ab} = p_b x_a - p_a x_b - \bar{\eta}\gamma_{ab}\eta, \quad (39)$$

where p_a is defined by expression in the first equation of motion given above. The Dirac quantization of this particle, after solving the second class constraints (and hence with the breaking of the Lorentz covariance in the spinorial sector of the phase space) gives the condition on states which singles out arbitrary spin and relates the mass and the spin

$$m_{J\pm} = \pm h \frac{J+1}{2} + \sqrt{h^2(J + \frac{1}{2})^2 + m_0^2} \quad (40)$$

The limit for the massless case can be considered as well and gives the description of particles with arbitrary helicity.

The second example of the arbitrary spin particle model comes [60] from the geometrical construction of the model on the six dimensional product space

of the Minkowski space M and the two-dimensional sphere S^2 . The family of Lagrangians of the model involves the joint interval in M and S^2 , with the metric on S^2 depending explicitly on fourvelocities. It is parametrized by the mass and spin (m, s) and has the following form

$$L = \frac{1}{2}e_1^{-1}(\dot{x}^2 - (e_1 mc)^2) + e_2^{-1}(4\frac{z\dot{\bar{x}}}{(\dot{x} \cdot \xi)^2}e_1^2 + (\Delta e_2)^2), \quad (41)$$

where z is a complex coordinate on the S^2 , e_1, e_2 are einbein fields associated with the reparametrizations in M and S^2 . The Δ is an additional ("spherical") mass $\Delta = \hbar mc \sqrt{s(s+1)}$ (s - spin). The conserved momentum tensor has the form

$$M_{ab} = x_a p_b - x_b p_a + (\bar{z}^{\dot{\alpha}}(\tilde{\sigma}_{ab})_{\dot{\alpha}\dot{\beta}}\bar{z}^{\dot{\beta}}p_{\bar{z}} - z^{\alpha}(\sigma_{ab})_{\alpha\beta}z^{\beta}p_z) \quad (42)$$

with the "internal" part defined by complex spherical coordinates. The Dirac quantization of this model gives, like in the previous one, the whole spectrum of spins. Let us note that this model exhibits the Zitterbevegung effect which is typical for the vectorial models.

3 Pseudoclassical models

In this section we pass to the models with the internal degrees of freedom described by the anti-commuting co-ordinates. The origins of the pseudomechanics should be dated back to the 1956, to the work of Martin [19]. However, anti-commuting variables appear firstly in the context of the functional integral for fermions in the works of Matthews, Salam [20] and Tobocman [21]. The extension of a configuration space to superspace enlarges the underlying symmetry group. Depending on the type of the model the extension of the Poincaré algebra yields the super-Poincaré algebra or some super algebra of the other kind. There are two types of such a models: vectorial and spinorial.

In models of vectorial type (the spinning particle models) the extension gives a untypical vectorial superalgebra, with the odd generators having vectorial index. Characteristic feature of such models is conventional character of the first quantized theory. Namely, the odd variables upon quantization are mapped into Dirac matrices and disappear on the quantum level. Such particle can be considered as a pseudoclassical limit of the conventional Dirac

quantum particle. In fact, this was the non-achieved goal of the vectorial classical models presented in the previous section.

The spinorial models (the superparticle models) have the super-Poincaré algebra (or its extension) as a symmetry generators. This models are connected more closely to the relativistic supersymmetry and the superparticles can be viewed upon, as a minimal, irreducible representations of the super-Poincaré group. However, such an objects contains the whole multiplet of fields with different spin but not only the spin one half component. On the first quantized level one still deals with the anti-commuting variables and instead of the wave functions the the wave super-functions have to be considered. The Dirac equation is not used literary but finds its superspace counterpart. It is worth noting that there exists an equivalence between some pseudoclassical and classical models of particles with spin which allows to generalize the notion of Zitterbevegung to the pseudoclassical case [15] (cf. as well Ref. [16]).

3.1 Spinning particles

Twenty years after the anti-commuting variables were introduced into the physical literature for the first time , there was proposed the spinning particle model by Berezin, Marinov [17] and Barducci, Casalbuoni, Lusanna [18].

The configuration space for this model is described by the set of co-ordinates $(x_\mu, \theta_\mu, \theta_5)$, where the θ -variables are anticommuting between themselves; θ_μ beeing fourvector and θ_5 a scalar. Proposed lagrangians were of the form

$$L_{BCL} = -m\sqrt{(\dot{x}^\mu - \frac{i}{m}\theta^\mu\dot{\theta}_5)(\dot{x}_\mu - \frac{i}{m}\theta_\mu\dot{\theta}_5) - \frac{i}{2}\theta_\mu\dot{\theta}^\mu - \frac{i}{2}\theta_5\dot{\theta}_5} \quad (43)$$

$$L_{BM} = -m\sqrt{-\dot{x}^2} + \frac{i}{2} \left(\theta_\mu\dot{\theta}^\mu + \theta_5\dot{\theta}_5 - \left(\frac{\dot{x}^\mu}{\sqrt{-\dot{x}^2}}\theta_\mu + \theta_5 \right) \lambda \right) \quad (44)$$

Let us focus on the model given by the first of above lagrangians. It is invariant under the supertranslations

$$x_\mu \mapsto x'_\mu = x_\mu - \epsilon_\mu A\theta_5 + \epsilon_5 B\theta_\mu \quad (45)$$

$$\theta_\mu \mapsto \theta'_\mu = \theta_\mu + \epsilon_\mu \quad (46)$$

$$\theta_5 \mapsto \theta'_5 = \theta_5 + \epsilon_5 \quad (47)$$

where A, B are numerical constants. The algebra of generators of these transformations is defined by the following relations

$$\{Q_\mu, Q_\nu\} = ag_{\mu\nu} \quad (48)$$

$$\{Q_5, Q_5\} = b \quad (49)$$

$$\{Q_\mu, Q_5\} = (B - C)P_\mu \quad (50)$$

The Q_μ, Q_5 commutes with P_μ . Performing canonical analysis of the model one gets the first class constraints

$$p^2 - m^2 = 0 \quad (51)$$

$$p_\mu \theta^\mu - m\theta_5 = 0 \quad (52)$$

After the first quantization one obtains precisely the Klein-Gordon and Dirac equations. For the θ -sector of the phase space the anticommutation relations

$$[\hat{\theta}_\mu, \hat{\theta}_\nu]_+ = -\hbar g_{\mu\nu} \quad (53)$$

$$[\hat{\theta}_5, \hat{\theta}_5]_+ = \hbar \quad (54)$$

$$[\hat{\theta}_\mu, \hat{\theta}_5]_+ = 0 \quad (55)$$

show that the classical variables originating from the Grassmann algebra are mapped after quantization to the elements of the appropriate Clifford algebra, here $\theta_\mu \mapsto \sqrt{\frac{\hbar}{2}}\gamma_\mu\gamma_5$, $\theta_5 \mapsto \sqrt{\frac{\hbar}{2}}\gamma_5$ (where γ_μ, γ_5 - Dirac matrices).

Above model can be generalized taking into account the reparametrization invariance, which yields the supergravity in $d=1$ [18] (cf. also J. van Holten's contribution to this volume). For the relativistic point particle one can explicitly achieve time reparametrization invariance by means of einbein field $e(\tau)$. In the case of the spinning particle it is necessary to introduce its supersymmetric partner $\psi(\tau)$. Resulting lagrangian takes the form

$$L = e^{-1}\dot{x}^2 + em^2 + i(\theta_\mu\dot{\theta}^\mu + \theta_5\dot{\theta}_5) + i(m\theta_5 - e^{-1}\dot{x}^\mu\dot{\theta}_\mu)\psi \quad (56)$$

The action given by this lagrangian is invariant under Poincaré transformations, reparametrizations and the local supersymmetry transformations, what justifies the association with the $D = 1$ supergravity. Namely,

$$\delta\tau = -\alpha(\tau) \quad (57)$$

$$\delta x^\mu = \alpha(\tau)\dot{x}^\mu + i\epsilon(\tau)\theta^\mu \quad (58)$$

$$\delta\theta^\mu = \alpha(\tau)\dot{\theta}^\mu + (2e)^{-1}\epsilon(\tau)(2\dot{x}^\mu - i\psi\theta^\mu) \quad (59)$$

$$\delta\theta_5 = \alpha(\tau)\dot{\theta}_5 - m\epsilon(\tau) \quad (60)$$

$$\delta e = \frac{d}{d\tau}(\alpha(\tau)e + i\epsilon(\tau)\psi) \quad (61)$$

$$\delta\psi = \frac{d}{d\tau}(\alpha(\tau)\psi + 2\epsilon(\tau)) \quad (62)$$

The Euler-Lagrange equations for the e and ψ are of algebraic character and this fields can be easily eliminated what yields the other version of the lagrangian which was considered in [28, 29]. The most general form of the action for the spinning particle with the supergravity multiplet can be given by the lagrangian of the form [35]

$$L = i g e^{-1} \dot{x}^2 - i b e m^2 + g \theta_\mu \dot{\theta}^\mu + b \theta_5 \dot{\theta}_5 + m b \theta_5 \psi \quad (63)$$

$$- (g e^{-1} + 2 e^{-3} \dot{x}^2 g') \psi (\dot{x}^\mu \dot{\theta}_\mu) + 2 e^{-2} g' (\dot{x}^\mu \dot{\theta}_\mu)^2 \quad (64)$$

All the spinning particle models have the property that they are the classical limits of the Dirac field theory and the anticommuting variables are present only in the classical description. The coupling of such models to the external electromagnetic or Yang-Mills fields yields vectorial superspace versions of the Bargmann-Michel-Telegdi or Wong equations [18]. Therefore in some sense the spinning particle models are improved versions of the conventional vectorial models, now with the proper quantum picture.

Let us finish this section with the superfield formulation of the spinning particle proposed by Ikemori [45, 46]. The first step consists in considering instead of single conventional time parameter a generalized super-time as a (1|1) dimensional superspace with coordinates (t, η) , where η is the new anticommuting variable. This means that trajectories of a system will take values in the superspace too. Namely,

$$X(t, \eta) = x(t) + i\eta\theta(t), \quad X \in C^\infty(t)[\eta]_0 \quad (65)$$

Above superfield unifies in one object the even and odd coordinates of the spinning particle. The supersymmetry present in the super-time space is called the little SUSY: $(t, \eta) \longrightarrow (t + \tau, \alpha\eta, \eta + \alpha)$, where τ , is an even and α an odd infinitesimal parameter. The algebra of supercharges and covariant derivatives is of the form

$$Q = \partial_\eta + \eta\partial_t, \quad \partial_\eta = \frac{\partial}{\partial\eta}, \quad \partial_t = \frac{\partial}{\partial t} \quad (66)$$

$$D = \partial_\eta - \eta \partial_t, \quad (67)$$

$$[Q, Q]_+ = 2\partial_t \quad (68)$$

$$[D, D]_+ = -2\partial_t \quad (69)$$

$$[Q, D]_+ = 0 \quad (70)$$

To introduce the local invariance the $d = 1$ supergravity multiplet is needed. It enters the super-zweibein field (E_A^M) , where $\partial_M = (\partial_t, \partial_\eta)$ and $\nabla_A = E_A^M \partial_M$. The action takes the form

$$S = \frac{1}{2} \int dt d\eta s \det(E_A^M) g^{AB} E_A^M \partial_M X^\mu E_B^N \partial_N X^\nu g_{\mu\nu}. \quad (71)$$

The customary choice of the gauge for the super-zweibein field is the following

$$(E_A^M) = \begin{pmatrix} E^{-1} & -e^{-1}\psi \\ -e^{-1}\eta & e^{-1}E \end{pmatrix}, \quad E = e + \eta\psi \quad (72)$$

Recently the model of the spinning particle with arbitrary number of supersymmetries on the world-line has been constructed [50, 51]. Such an N -extended little SUSY in the massive model of the spinning particle, after the field redefinitions in the equations of motion, yields the supersymmetric Lax equation. Moreover it can be used in the study of hyperbolic Kac-Moody algebras.

3.2 Superparticle models

The extension of the Minkowski space to the superspace with the additional spinorial coordinate is the basic structure of the supersymmetric field theories [44].

The super-Poincaré group becomes the fundamental symmetry of the theory. Now, the superparticles are generalizations of the relativistic point particle from the Minkowski space to such a superspace, with still the same requirement at the background - to describe "correctly" the spin. Because they incorporate super-Poincaré invariance there is a close connection between superparticles and representation of supersymmetry. Some models provide a natural examples of actions which yield, after the first quantization, the minimal irreducible representations of the given super-Poincaré superalgebra.

The superparticles have reach symmetry, as local as rigid [30]. In many respects the quantization procedure is difficult because of the complicated

structure of the phase space constraints. This aspect of the superparticle models makes that they are instructive toy models used to understand the superstrings and the variety of their quantization procedures.

There are important differences between massless and massive models, however we will not stress them, aiming only to illustrate generally the historical development in the construction of the models.

The first pseudoclassical relativistic particle model with the spinorial grassmanian co-ordinates was proposed by Casalbuoni in 1976 [13]. On the configuration superspace $(x_\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$ he defined the lagrangian of the form

$$L = -m\sqrt{\dot{\omega}_\mu \dot{\omega}^\mu} \quad (73)$$

where

$$d\omega_\mu = dx_\mu - i(d\theta\sigma_\mu\bar{\theta} - \theta\sigma_\mu d\bar{\theta}) \quad (74)$$

is the super one-form, invariant under supertranslations

$$x_\mu \mapsto x'_\mu = x_\mu - i(\epsilon\sigma_\mu\bar{\theta} - \theta\sigma_\mu\bar{\epsilon}) \quad (75)$$

$$\theta_\alpha \mapsto \theta'_\alpha = \theta_\alpha + \epsilon_\alpha \quad (76)$$

$$\bar{\theta}_{\dot{\alpha}} \mapsto \bar{\theta}'_{\dot{\alpha}} = \bar{\theta}_{\dot{\alpha}} + \bar{\epsilon}_{\dot{\alpha}} \quad (77)$$

The lagrangian is too poor to give after the first quantization the Dirac equations and some interesting supersymmetric multiplets. The chiral supermultiplet content of the Casalbuoni's G_4 model was analysed by Almond [14]. To improve this model the first order fermionic kinetic terms are needed. But they cannot be introduced in a straightforward way, because of the relation

$$\theta_\alpha \frac{d}{d\tau} \theta^\alpha = \frac{d}{d\tau} (\theta_\alpha \theta^\alpha) \quad (78)$$

Four years later Volkov and Pashnev [22] tried to cure this drawback using more general super one-form, invariant under super-Poincaré transformations. Namely,

$$ds^2 = d\omega_\mu d\omega^\mu + a d\theta^\alpha d\theta_\alpha - a^* d\bar{\theta}^{\dot{\alpha}} d\bar{\theta}_{\dot{\alpha}}, \quad a \in C \quad (79)$$

and then the action of the form

$$S = -m \int_{\tau_1}^{\tau_2} \sqrt{ds^2} = -m \int_{\tau_1}^{\tau_2} \sqrt{\dot{\omega}_\mu \dot{\omega}^\mu + a \dot{\theta}^\alpha \dot{\theta}_\alpha - a^* \dot{\bar{\theta}}^{\dot{\alpha}} \dot{\bar{\theta}}_{\dot{\alpha}}} \tau \quad (80)$$

Now the fermionic kinetic term is present and gives the first class constraints. However, not the one playing upon quantization the rôle of the Dirac equation. Nevertheless the content of the model is more reach, since the first quantized theory contains some multiplets (two scalar multiplets and one vector multiplet of states with the negative norm).

The Brink-Schwarz action for a superparticle of mass m in d dimensions uses again invariant super oneform ω . The reparametrization invariance is provided by the einbein field what enables to consider a massless superparticle as well. In 1981 they proposed an action of the form [27]

$$S = \int d\tau (e^{-1}\omega^2 - em) \quad (81)$$

where

$$\omega^n = \dot{x}^n + i\dot{\theta}\Gamma^n\theta, \quad n = 1, 2, \dots, d-1 \quad (82)$$

Specialy massless case is intersting here, because there is an additional invariance present (Siegel [33])

$$\delta_\kappa \theta^\alpha = \varpi^{\alpha\beta} \kappa_\beta, \quad \varpi^{\alpha\beta} = \omega_n (\Gamma^n)^{\alpha\beta} \quad (83)$$

$$\delta_\kappa x^n = -i\delta_\kappa \theta \Gamma^n \theta \quad (84)$$

$$\delta_\kappa e = 4ie\dot{\theta}\kappa \quad (85)$$

whre κ is an anticommuting spinoral parameter. This symmetry allows to reduce some of the θ - degrees of freedom (here half of them, in general at most half) [32, 33, 34].

The first massive superparticle model which exhibits κ - symmetry was introduced in 1982 by de Azcárraga and Lukierski [23]. In their model this symmetry was firstly observed but the rôle of such a gauge invariance in reduction of the degrees of freedom was first pointed out by Siegel [33] and he introduced modified action. To finally overcome the problem with the fermionic kinetic term present in the Casalbuoni's model one has to enlarge the superspace. In the de Azcárraga-Lukierski model it is done by considering the N -extended Minkowski superspace $(x_\mu, \theta_{i\alpha}, \bar{\theta}_i^{\dot{\alpha}})$, $i = 1, 2, \dots, N$ and introducing central charges to the superalgebra. Hence the resulting underlying rigid symmetry gets enlarged to N -extended super-Poincaré superalgebra.

The new "isotopic" structure allows to use internal symplectic metric $A_{ij} = -A_{ji}$ and the expression of the form $\theta^{i\alpha} A_{ij} \frac{d}{d\tau} \theta^{j\alpha}$ now is not a total time derivative and can contribute nontrivially to the action. After obvious modification in the super one form

$$d\omega_\mu = dx_\mu - i(d\theta_i \sigma_\mu \bar{\theta}_i - \theta_i \sigma_\mu d\bar{\theta}_i) \quad (86)$$

the lagrangian function can be written as

$$L = -m\sqrt{\dot{\omega}_\mu\dot{\omega}^\mu} + i(\theta_\alpha^i A_{ij}\dot{\theta}^{j\alpha} + \bar{\theta}_{\dot{\alpha}}^i A_{ij}\dot{\bar{\theta}}^{j\dot{\alpha}}) \quad (87)$$

The fermionic kinetic term is in fact of the Wess-Zumino type and changes under supersymmetry transformations by a total time derivative. Indeed, let Z_{IJ} be a symmetric, Lorentz invariant matrix (where I, J could be multi-indices e.g. $I = (\alpha, i)$ and $Z_{IJ} = \epsilon_{\alpha\beta}A_{ij}$), then the simple example of the WZ-term for a supersymmetric particle is of the form

$$S_{WZ} = \int d\tau i\theta^I Z_{IJ}\theta^J \quad (88)$$

What means that one starts from the closed super-twoform $h = id\theta Z d\theta$. It is exact; with $b = id\theta Z\theta$ one can write $h = db$. From the invariance of h under supertranslations it follows that $d(\delta_\epsilon b) = 0$ and at least locally $\delta_\epsilon b = df$ for some superfunction f . For the AL-action it means that ϵ -variation yields the total time derivative change in the lagrangian.

The AL-model after the first quantization yields the irreducible representations of the N -extended super-Poincaré superalgebra. The whole spectrum of supersymmetric multiplets was found as a result of the first quantization [77, 39] not only for the massive case but also for the massless [40]. In the quantization of this model there was firstly applied the supersymmetric generalization of the Gupta-Bleuler [36, 37] quantization method [38, 39], which later was used in quantization of various systems exhibiting the similar structure of the second class constraints (i.e. hermitean splitting of the set of the second class constraints into the subsets of conjugated, relatively first class constraints).

The coupling of this model to the external fields gives interesting results. Comparing to the traditional equations of the spin precession in the external electromagnetic field we obtain that the superspace generalization of the Bargmann-Michel-Telegdi equations takes the form [41]

$$\dot{p}^\mu = eF^{\mu\nu}\dot{x}_\nu + gS_{\rho\lambda}^{Spin}\partial^\mu F^{\rho\lambda} \quad (89)$$

$$\frac{d}{d\tau}W_\mu = gF_{\mu\nu}W^\nu + (\frac{e}{2m} - g)\dot{\omega}_\nu F^{\nu\rho}W_\rho\dot{\omega}_\mu - \quad (90)$$

$$-\frac{e}{2m}(W_\nu\dot{\omega}_\mu - W_\mu\dot{\omega}_\nu)F^{\nu\rho}\dot{z}_\rho, \quad (91)$$

where $W_\mu = \frac{i}{2m^2}\epsilon_{\mu\nu\rho\lambda}p^\nu S_{Spin}^{\rho\lambda}$ and $S_{\mu\nu}^{Spin} = \frac{i}{2}\epsilon_{\mu\nu\rho\lambda}p^\rho(i\theta_k\sigma^\lambda\bar{\theta}_k)$. For the external Yang-Mills field we obtain, within the minimal coupling, the generalized

Wong equations [41]

$$\dot{p}^\mu = g_1 F_a^{\mu\nu} I^a \dot{x}_\nu + g_2 S_{\rho\lambda}^{Spin} \partial^\mu F_a^{\rho\lambda} I^a \quad (92)$$

$$\dot{I}^a = g_1 f_{bc}^a A_\mu^b I^c \dot{x}^\mu - g_2 f_{bc}^a F_{\mu\nu}^b I^c S_{Spin}^{\mu\nu} \quad (93)$$

$$\frac{d}{d\tau} W_\mu = g_2 F_{\mu\nu}^a W^\nu I_a + \left(\frac{g_1}{2m} - g_2 \right) \dot{\omega}_\nu F_a^{\nu\rho} W_\rho \dot{\omega}_\mu I^a - \quad (94)$$

$$- \frac{g_1}{2m} (W_\nu \dot{\omega}_\mu - W_\mu \dot{\omega}_\nu) F_a^{\nu\rho} \dot{z}_\rho I^a, \quad (95)$$

However, the AL-model is supersymmetric therefore the fully supersymmetric coupling to the supersymmetric field is of greater interest. It can be found in Ref. [56]. In the case of the supersymmetric Yang-Mills and supergravity theories it gives in a natural way the conventional sets of constraints for these fields.

3.3 Twistorial models

The supersymmetric particle models using the twistor-like variables were developed in the eighties, firstly in the component formulation then in the superfield one [57, 58, 61, 62, 67, 68]. The very important result obtained within this formulation consists in re-expressing upon use of the equations of motion the local world-line supersymmetry as κ -transformation [67, 68]. General feature of this kind of models is a possibility of manifestly covariant quantization. To merely signal the existence of very reach developments let us recall the superfield version of the model beeing generalization of the following component action [67]

$$S = \int d\tau p_m (\dot{x}^m - i\bar{\theta}\gamma^m\theta + \bar{\lambda}\gamma^m\lambda). \quad (96)$$

Namely,

$$S_1 = -i \int d\tau d\eta P_m (DX^m + i\bar{\Theta}\gamma^m D\Theta), \quad (97)$$

where $D = \partial_\eta + i\eta\partial_\tau$ and $P_m = p_m + i\eta\rho_m$, $X_m = x_m + i\eta\chi_m$, $\Theta_\alpha = \theta_{alpha} + \eta\lambda_\alpha$. In the component version this action contains additional to the S an auxiliary term of the form

$$S_a = i \int d\tau \rho_m (\chi^m + \bar{\theta}\gamma^m\lambda) \quad (98)$$

The mechanism of trading the twistorial superparticle's κ -symmetry for world-line supersymmetry is analysed in series of papers [67, 68, 69] and recently

in Ref. [70]. Relation between the different forms of the superparticle dynamics, involving spinorial coordinates is analysed in Ref. [71].

The possibility of manifestly covariant quantization of the massless particle model was the motivation of development of the model of harmonic superparticle [72]. The action of this model is a generalization of the Siegel model with some new (harmonic) bosonic variables which are parametrising a suitably chosen coset spaces.

3.4 Arbitrary superspin models

The model of the classical arbitrary spin particle [60] discussed in Sec.2.2. can be generalized to the pseudoclassical model with the N-extended super-Poincaré symmetry [66]. After the Dirac quantization this model gives the on-shell massive chiral superfields (the central charges can be introduced as well).

The extension of the configuration space $M \times S^2$ is done in the Minkowski sector, it is changed into the N-extended superMinkowski superspace with coordinates $(x^a, \theta^{\alpha I}, \bar{\theta}_{\dot{\alpha} I})$ $I = 1, 2, \dots, N$, $a = 0, 1, 2, 3$. On the new configuration space $M^{4|4N} \times S^2$ there is defined the Lagrangian of the form

$$L = \frac{1}{2}e_1^{-1}(\Pi^2 - (e_1 m)^2) + e_2^{-1}(4\frac{z\dot{x}}{(\Pi \cdot \xi)^2}e_1^2 + (\Delta e_2)^2), \quad (99)$$

where

$$\Pi^a = \dot{x}^a + i\theta^I \sigma^a \dot{\bar{\theta}}_I - \dot{\theta}^I \sigma^a \theta_I \quad (100)$$

$$\xi_a = (\sigma_a)_{\alpha\dot{\beta}} z^\alpha \bar{z}^{\dot{\beta}} \quad (101)$$

$$\Delta = m\sqrt{Y(Y+1)} \quad (102)$$

The Y is a superspin parameter. The central charges analogous to those of the Azcárraga-Lukierski model can be considered here as well [66].

4 Doubly supersymmetric models

The doubly supersymmetric models were considered firstly by Gates and Nishino [47]. To extend the NSR string theory they proposed a new class of superstring models which possess both spacetime and world-sheet supersymmetries. Then within this scheme the particle model was considered [48, 49].

There are also two other approaches to such particle models: the twistor-like superfield models (commented in previous section) and the spinning particle model (invented firstly in the component form [73, 74]).

The spinning particle models are reviewed in Ref. [78], here we shall restrict ourselves to the brief illustration of the superfield realization [75, 76]. In the (supersymmetry)² particle models one introduces the supertime space (t, η) and the superMinkowski superspace. Therefore the trajectories of a point object are the mappings

$$(t, \eta) \in T \mapsto M_D \ni (X^m, \Theta^A) \quad (103)$$

where

$$X^m(t, \eta) = x^m(t) + \eta \Lambda^m(t) \quad (104)$$

$$\Theta^A(t, \eta) = \theta^A(t) + \eta \varphi^A \quad (105)$$

Introducing the covariant object

$$Y^m = \nabla_\eta X^m + i \Theta \gamma^m \nabla_\eta \Theta, \quad (106)$$

where ∇ is the covariant superderivative given by $\nabla_A = E_A^M \partial_M$ (cf. eq. (71)) one can write the supersymmetric invariant action in the form

$$S = \frac{1}{2} \int dt d\eta s \det(E_A^M) \nabla_\eta Y^m \cdot Y_m = \quad (107)$$

$$= \frac{1}{2} \int dt (e^{-1} \dot{\omega}^2 - 2e^{-1} \psi \lambda^m \dot{\omega}_m - 2i(\dot{\omega}^m - \psi \lambda^m) \bar{\varphi} \gamma_m \varphi - \quad (108)$$

$$-e(\varphi \gamma^m \varphi)^2 + \lambda \dot{\lambda}) \quad (109)$$

The superfield covariant phase space description of this model in the rigid supersymmetry case was given in Ref. [77].

One can say that developments in the spinning and superparticle models has been resumed in their superfield formulation which appeared in the second half of the eighties. It has turned out that all types of the pseudomechanical description can be put together and organized in a joint superfield model (let us note that there exists the superfield formulation of a spinning particle alone, but not of the superparticle, which has to coexist in the superfield formulation with the spinning particle).

5 Recent developments: q-deformed spinning particle and κ -relativistic model

Finally let us mention the brand new aspect of the relativistic particle models, namely their deformations. Actually there are not well established deformed models. However, without entering into the question why things have to be (or not to be) deformed we shall recall two examples: the q-deformed spinning particle and the κ -relativistic particle.

The example of the q-deformed relativistic spinning particle was considered by Malik [79]. With the use of the first order Lagrangian of the spinning particle and the q-deformed graded commutation relations for (x^m, ψ^m, p^m, e) in the phase space he introduces "deformed" $GL_q(2)$ -invariant Lagrangian

$$L = \sqrt{q} p_m \dot{x}^m + \frac{i}{2} \psi^m \dot{\psi}_m - \frac{e}{1+q^2} p^2 + i\chi \psi_m p^m \quad (110)$$

The model is under investigation and its Dirac "deformed" quantization is still to be performed.

The κ -relativistic particle is more "physical". It lives in the κ -deformed Minkowski space [82, 83] with the mass shell condition modified to the following form

$$(2\kappa \sinh \frac{p_0}{2\kappa})^2 - \vec{p}^2 = m^2 \quad (111)$$

This model can be described within the formalism with commuting as well as noncommuting space-time coordinates. The interesting properties of the object of this kind are discussed in Refs. [80, 81].

Acknowledgements

This work is supported in part by KBN Grant # 2 P302 087 06.

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January 1996, ITP UWr 901/96